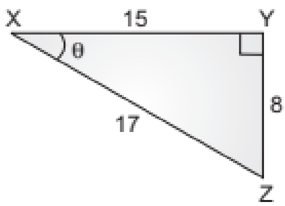
OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Consider the triangle shown below.



What are the values of tan θ, cosec θ and sec θ?

(a) $\tan\theta = \frac{8}{15}$, $\text{cosec}\,\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{8}$

(b) $\tan\theta = \frac{8}{15}$, $\text{cosec}\,\theta = \frac{17}{8}$, $\sec\theta = \frac{17}{15}$

(c) $\tan\theta = \frac{17}{15}$, $\text{cosec}\,\theta = \frac{8}{15}$, $\sec\theta = \frac{17}{8}$

(d) $\tan\theta = \frac{8}{15}$, $\text{cosec}\,\theta = \frac{17}{15}$, $\sec\theta = \frac{8}{17}$

[CBSE Question Bank 2022]

Ans. (d) $5\frac{1}{3}$

2. If $x \tan 60° \cos 60° = \sin 60° \cot 60°$, then $x =$

(a) cos 30° (b) tan 30°
(c) sin 30° (d) cot 30°

[CBSE SQP Std. 2022]

Ans. (b) tan30°

[CBSE Marking Scheme SQP Std. 2022]

Explanation: $x \tan 60° \cos 60° = \sin 60° \cot 60°$

$$(x) \times (\sqrt{3}) \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{3}x}{2} = \frac{1}{2}$$

$$x = \frac{1}{\sqrt{3}} = \tan 30°$$

3. If $\cot\theta = \frac{1}{\sqrt{3}}$, then value of $\sec^2\theta + \text{cosec}^2\theta$ is:

(a) 1 (b) $\frac{40}{9}$

(c) $\frac{38}{9}$ (d) $5\frac{1}{3}$

[CBSE Term-1 Std. 2021]

Explanation:

$$\cot\theta = \frac{1}{\sqrt{3}} = \cot 60° \quad \left[\because \cot 60° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \theta = 60°$$

Now, $\sec^2\theta + \text{cosec}^2\theta = \sec^2 60° + \text{cosec}^2 60°$

$$= (2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 4 + \frac{4}{3}$$

$$= \frac{12+4}{3} = \frac{16}{3} = 5\frac{1}{3}$$

4. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 0°)$ then the value of tan θ is:

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

Ans. (a) $\sqrt{2} - 1$

Explanation: We have,

$$\sin\theta + \cos\theta = \sqrt{2}\cos\theta$$

$$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \sqrt{2} - 1$$

$$\Rightarrow \tan\theta = \sqrt{2} - 1$$

5. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$

(a) 1 (b) 2
(c) 3 (d) 4

[CBSE SQP Std. 2022]

Ans. *(b) 2*

[CBSE Marking Scheme SQP Std. 2022]

Explanation: Given, $\sin\theta + \cos\theta = \sqrt{2}$

On squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

$$2\sin\theta\cos\theta = 2 - 1 = 1$$

$$\frac{1}{\sin\theta\cos\theta} = 2 \quad \text{...(i)}$$

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\tan\theta + \cot\theta = 2$$

6. The sine of an angle in a right triangle is $\frac{4}{5}$. Which of these could be the measures of the sides of the triangle?

(a) 4 cm, 5 cm and 9 cm

(b) 4 cm, 5 cm and $\sqrt{41}$ cm

(c) 6 cm, 8 cm and 10 cm

(d) 8 cm, 10 cm and $4\sqrt{41}$ cm

[CBSE Question Bank 2023]

Ans. *(c) 6 cm, 8 cm and 10 cm*

Explanation: Given, sine of an angle = $\frac{4}{5}$

Let θ be the angle of the right triangle.

By Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

where, hypotenuse is the largest side among all.

So, (a) 4 cm, 5 cm and 9 cm

$$(9)^2 = 81 \text{ cm}$$

$$(4)^2 + (5)^2 = 16 + 25 = 41 \text{ cm}$$

LHS ≠ RHS.

(b) 4 cm, 5 cm and $\sqrt{41}$ cm

$$(\sqrt{41})^2 = 41 \text{ cm}$$

$$(4)^2 + (5)^2 = 16 + 25 = 41 \text{ cm}$$

LHS = RHS

But, $\sin\theta = \frac{4}{5}$ (given)

Here, perpendicular = 4 cm, while hypotenuse = 5 cm.

Thus, it is not as per the condition.

(c) 6 cm, 8 cm and 10 cm

$$(10)^2 = 100 \text{ cm}$$

$$(6)^2 + (8)^2 = 36 + 64 = 100 \text{ cm}$$

LHS = RHS

But, $\sin\theta = \frac{4}{5}$ (given)

Here, perpendicular = 4 cm and hypotenuse = 5 cm

Thus, it is as per the condition, here, perpendicular = 8 cm, hypotenuse = 10 cm and base = 6 cm.

(d) 8 cm, 10 cm and $4\sqrt{41}$ cm

$$(4\sqrt{41})^2 = 656$$

$$(8)^2 + (10)^2 = 164$$

LHS ≠ RHS

7. If sin (A + B) = cos (A – B) = 1, then:

(a) A = B = 0 **(b) A = B = 45°**

(c) A = 60°, B = 30° **(d) A = 30°, B = 60°**

8. If $\cos A = \frac{5}{13}$, then the value of $\tan A + \cot A$ is:

(a) $\frac{169}{60}$ **(b) $\frac{12}{13}$**

(c) 1 **(d) $\frac{60}{169}$**

9. If $5\tan\beta = 4$, then $\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$

(a) $\frac{1}{3}$ **(b) $\frac{2}{5}$**

(c) $\frac{3}{5}$ **(d) 6**

[CBSE SQP Std. 2022]

Ans. *(a)* $\frac{1}{3}$

[CBSE Marking Scheme SQP Std. 2022]

Explanation: Given,

$$5 \tan \beta = 4$$

or $$\frac{\sin\beta}{\cos\beta} = \frac{4}{5}$$

$\therefore$ $$\sin \beta = \frac{4}{5}\cos\beta$$

The given trigonometric expression is,

$$\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta}$$

$$\left[\text{Substituting } \sin\beta = \frac{4}{5}\cos\beta\right]$$

$$= \frac{5\times\frac{4}{5}\cos\beta - 2\cos\beta}{5\times\frac{4}{5}\cos\beta + 2\cos\beta}$$

$$= \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$$

10. **If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, then the value of $5\left(x^2 - \frac{1}{x^2}\right)$ is:**

(a) 5 **(b)** $\frac{1}{5}$

(c) $\frac{2}{5}$ **(d)** 0

Ans. *(b)* $\frac{1}{5}$

Explanation: Given, $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$

Squaring both sides, we get

$$25x^2 = \sec^2\theta \quad ...(i)$$

$$\frac{25}{x^2} = \tan^2\theta \quad ...(ii)$$

Subtracting the two equations, we get

$$\Rightarrow \quad 25x^2 - \frac{25}{x^2} = \sec^2\theta - \tan^2\theta$$

$$\Rightarrow \quad 25\left(x^2 - \frac{1}{x^2}\right) = 1 \quad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$\Rightarrow \quad 5\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{5}$$

11. **If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is:**

(a) 1 **(b)** $\frac{1}{2}$

(c) 2 **(d)** 3

[NCERT Exemplar]

Ans. *(a)* 1

Explanation: We know that,

$$\sin^2\theta + \cos^2\theta = 1 \quad ....(i)$$

Given, $\sin A + \sin^2 A = 1$

$$\Rightarrow \quad \sin A = 1 - \sin^2 A$$

$$\Rightarrow \quad \sin A = \cos^2 A \quad \text{[Using (i)]}$$

Squaring both sides, we get

$$\Rightarrow \quad \sin^2 A = \cos^4 A$$

$$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A \quad \text{[Using (i)]}$$

$$\Rightarrow \quad \cos^2 A + \cos^4 A = 1$$

12. **If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0° < A + B \le 90°$ and $A > B$, then find the measures of angles A and B.**

(a) 30°, 45° **(b)** 60°, 30°

(c) 45°, 90° **(c)** 30°, 90°

[CBSE SQP Std. 2022]

13. **The value of $(1 + \cos\theta)(1 - \cos\theta)\text{cosec}^2\theta$ is:**

(a) 0 **(b)** 1

(c) $\cos^2\theta$ **(d)** $\sin^2\theta$

Ans. *(b)* 1

Explanation: $(1 + \cos\theta)(1 - \cos\theta)\text{cosec}^2\theta$

$$= (1 - \cos^2\theta)\text{cosec}^2\theta$$

$$= \sin^2\theta\, \text{cosec}^2\theta$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \sin^2\theta \times \frac{1}{\sin^2\theta}$$

$$\left[\because \text{cosec}\,\theta = \frac{1}{\sin\theta}\right]$$

$$= 1$$

14. **If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is:**

(a) 1 **(b)** $\frac{3}{4}$

(c) $\frac{1}{2}$ **(d)** $\frac{1}{4}$ **[NCERT Exemplar]**

Ans. *(c)* $\frac{1}{2}$

Explanation:

Given, $(\sin\theta - \cos\theta) = 0$

$$\Rightarrow \quad \sin\theta = \cos\theta$$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \quad \tan\theta = 1 \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \quad \tan\theta = \tan 45° \quad [\because \tan 45° = 1]$

$\Rightarrow \quad \theta = 45°$

Now,

$$\sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$

$$\left[\because \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

15. Find an acute angle θ when:

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

(a) 60° (b) 30°

(c) 90° (c) 45°

[CBSE SQP Std. 2022]

Ans. *(a) 60°*

Explanation: $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Dividing the numerator and denominator of LHS by cosθ, we get

$$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Which on simplification (or comparison) gives

$$\tan\theta = \sqrt{3}$$

Or $\theta = 60°$

[CBSE Marking Scheme SQP Std. 2022]

16. If θ = 45°, then the value of sec θ cot θ – cosec θ tan θ is:

(a) 0 (b) 1

(c) 2 (d) 3

17. If $x = a \sin\theta$ and $y = a\cos\theta$, then the value of $x^2 + y^2$ is:

(a) a (b) a^2

(c) 1 (d) b^2

Ans. *(b) a^2*

Explanation: Given, $x = a\sin\theta$ and $y = a\cos\theta$

Squaring both sides, we get

$x^2 = a^2\sin^2\theta$ ...(i)

$y^2 = a^2\cos^2\theta$ ...(ii)

Adding the two equations, we get

$$x^2 + y^2 = a^2\sin^2\theta + a^2\cos^2\theta$$
$$= a^2(\sin^2\theta + \cos^2\theta)$$
$$= a^2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

***18. If $\sin A = \frac{1}{2}$, then the value of sec A is:**

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) 1

[CBSE SQP Basic 2022]

Ans. *(a)* $\frac{2}{\sqrt{3}}$

[CBSE Marking Scheme SQP Basic 2022]

Explanation: $\sin A = \frac{1}{2}$

Hence, $A = 30°$

Therefore, $\sec A = \sec 30°$

$$= \frac{2}{\sqrt{3}}$$

19. $4\tan^2 A - 4\sec^2 A$ is equal to:

(a) 1 (b) –1

(c) 4 (d) –4

Ans. *(d) –4*

Explanation: $4\tan^2 A - 4\sec^2 A$

$$= -4(\sec^2 A - \tan^2 A)$$
$$= -4 \times 1$$
$$[\because \sec^2\theta - \tan^2\theta = 1]$$
$$= -4$$

20. If 3 cos θ = 1, then cosec θ is equal to:

(a) $2\sqrt{2}$ (b) $\frac{3}{2\sqrt{2}}$

(c) $\frac{2\sqrt{3}}{3}$ (d) $\frac{4}{3\sqrt{2}}$

21. $\sqrt{3}\cos^2 A + \sqrt{3}\sin^2 A$ is equal to:

(a) 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) 0

[CBSE SQP Basic 2022]

* Answer given by CBSE is wrong. The correct answer is $\frac{2}{\sqrt{3}}$.

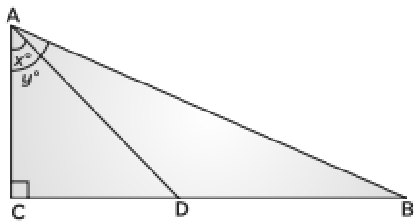
Ans. *(c)* $\sqrt{3}$

[CBSE Marking Scheme SQP Basic 2022]

Explanation: $\sqrt{3}\cos^2A + \sqrt{3}\sin^2A$

$$= \sqrt{3}(\cos^2A + \sin^2A)$$
$$= \sqrt{3} \times 1 \quad [\because \cos^2A + \sin^2A = 1]$$
$$= \sqrt{3}$$

22. In the given figure, D is the mid-point of BC, then the value of $\frac{\cot y°}{\cot x°}$ is:



(a) 2 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

[CBSE Term-1 SQP 2021]

Ans. *(b)* $\frac{1}{2}$

$$\frac{\cot y°}{\cot x°} = \frac{AC/BC}{AC/CD} = \frac{CD}{BC} = \frac{CD}{2CD} = \frac{1}{2}$$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: In ΔADC,

$$\cot x° = \frac{AC}{CD} \quad ...(i)$$

and, in ΔABC,

$$\cot y° = \frac{AC}{BC} = \frac{AC}{2CD} \quad ...(ii)$$

[$\because$ D is mid-point of BC]

So, $$\frac{\cot y°}{\cot x°} = \frac{\frac{AC}{2CD}}{\frac{AC}{CD}}$$
$$= \frac{1}{2}$$

23. If cosec θ – cot θ = $\frac{1}{3}$, then the value of cosec θ + cot θ is:

(a) 1 (b) 2

(c) 3 (d) 4

24. The value of cos 1° cos 2° cos 3° cos 4° .... cos 90° is:

(a) 1 (b) 0

(c) –1 (d) 2

[CBSE SQP Basic 2022]

Ans. *(b)* *0*

[CBSE Marking Scheme SQP Basic 2022]

Explanation: cos 1° cos 2° cos 3° ... cos 90°

$\therefore$ (cos 1°) × (cos 2°) × (cos 3°) × ... × (cos 89°) × (cos 90°)

cos 90° = 0

$\therefore$ (cos 1°) × (cos 2°) × (cos 3°) × ...× (cos 89°) × (cos 90°) = 0

25. If $2\sin 2\theta = \sqrt{3}$, then the value of θ is:

(a) 90° (b) 30°

(c) 60° (d) 45°

26. Given that $\sec\theta = \sqrt{2}$, then the value of $\frac{1+\tan\theta}{\sin\theta}$ is:

(a) $2\sqrt{2}$ (b) $\sqrt{2}$

(c) $3\sqrt{2}$ (d) 2

[CBSE Term-1 Std. 2021]

Ans. *(a)* $2\sqrt{2}$

Explanation:

$$\sec\theta = \sqrt{2} = \sec 45°$$
$$\Rightarrow \quad \theta = 45°$$

Now, $$\frac{1+\tan\theta}{\sin\theta} = \frac{1+\tan 45°}{\sin 45°}$$
$$= \frac{1+1}{\frac{1}{\sqrt{2}}} = 2\sqrt{2}$$

27. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is:

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$

[CBSE Term-1 Std. 2021]

Ans. *(c)* $\frac{\sqrt{2}}{2}$

Explanation: We have,

$$\tan\theta + \cot\theta = 2$$

$\Rightarrow \quad \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \quad (\tan\theta - 1)^2 = 0$

$\Rightarrow \quad \tan\theta - 1 = 0$

$\Rightarrow \quad \tan\theta = 1 = \tan 45°$

$\Rightarrow \quad \theta = 45°$

Now, $\sin^3\theta + \cos^3\theta = \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3$

$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$, or $\frac{\sqrt{2}}{2}$

28. **In ΔABC right-angled at B, $\sin A = \frac{7}{25}$, then the value of $\cos C$ is:**

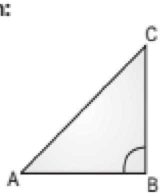
(a) $\frac{7}{25}$ (b) $\frac{24}{25}$

(c) $\frac{7}{24}$ (d) $\frac{24}{7}$

[CBSE Term-1 Std. 2021]

Ans. *(a)* $\frac{7}{25}$

Explanation:



We have,

$\sin A = \frac{7}{25} = \frac{BC}{AC}$

Now, $\cos C = \frac{BC}{AC} = \frac{7}{25}$

29. **If $\tan\alpha + \cot\alpha = 2$, then $\tan^{20}\alpha + \cot^{20}\alpha$ equal to:**

(a) 0 (b) 2

(c) 20 (d) 220

[CBSE Term-1 SQP 2021]

30. **If $1 + \sin^2\alpha = 3\sin\alpha\cos\alpha$, then values of $\cot\alpha$ are:**

(a) –1, 1 (b) 0, 1

(c) 1, 2 (d) –1, –1

[CBSE Term-1 SQP 2021]

Ans. *(c) 1, 2*

$1 + \sin^2\alpha = 3\sin\alpha\cos\alpha$

$\sin^2\alpha + \cos^2\alpha + \sin^2\alpha = 3\sin\alpha\cos\alpha$

$2\sin^2\alpha - 3\sin\alpha\cos\alpha + \cos^2\alpha = 0$

$(2\sin\alpha - \cos\alpha)(\sin\alpha - \cos\alpha) = 0$

$\therefore \quad \cot\alpha = 2$ or $\cot\alpha = 1$

[CBSE Marking Scheme Term-1 SQP 2021]

31. **If $2\sin^2\beta - \cos^2\beta = 2$, then β is:**

(a) 0° (b) 90°

(c) 45° (d) 30°

[CBSE Term-1 SQP 2021]

Ans. *(b) 90°*

$2\sin^2\beta - \cos^2\beta = 2$

Then, $2\sin^2\beta - (1 - \sin^2\beta) = 2$

$3\sin^2\beta = 3$ or $\sin^2\beta = 1$

β is 90°.

[CBSE Marking Scheme Term-1 SQP 2021]

32. **If the angles of ΔABC are in the ratio 1: 1: 2, respectively (the largest angle being angle C), then the value of $\frac{\sec A}{\operatorname{cosec} B} - \frac{\tan A}{\cot B}$ is:**

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{\sqrt{3}}{2}$

[CBSE Term-1 SQP 2021]

Ans. *(a) 0*

$1x + 1x + 2x = 180°, x = 45°$

$\angle A, \angle B$ and $\angle C$ are 45°, 45° and 90° respectively.

$\frac{\sec A}{\operatorname{cosec} B} - \frac{\tan A}{\cot B} = \frac{\sec 45°}{\operatorname{cosec} 45°} - \frac{\tan 45°}{\cot 45°}$

$= \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{1} = 1 - 1 = 0$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: Given, in ΔABC

$\angle A : \angle B : \angle C = 1 : 1 : 2$

Let, $\angle A = x, \angle B = x$ and $\angle C = 2x$

We know,

$\angle A + \angle B + \angle C = 180°$

(Angle sum property of triangle)

$\therefore \quad x + x + 2x = 180°$

$\Rightarrow \quad 4x = 180°$

$\Rightarrow \quad x = 45°$

$\therefore \angle A = \angle B = 45°$ and $\angle C = 2 \times 45° = 90°$

Now, $\frac{\sec A}{\operatorname{cosec} B} - \frac{\tan A}{\cot B}$

$$= \frac{\sec 45°}{\text{cosec} 45°} - \frac{\tan 45°}{\cot 45°}$$

$$= \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{1}$$

$$= 1 - 1 = 0$$

33. If $\sec\theta + \tan\theta + 1 = 0$, then $\sec\theta - \tan\theta$ is:

(a) –1 (b) 1

(c) 0 (d) 2 **[CBSE 2013]**

34. Given that $\tan\theta = \frac{1}{\sqrt{3}}$, then the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is:

(a) – 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ **[CBSE 2013]**

35. $\frac{2\tan 30°}{1+\tan^2 30°} =$

(a) sin 30° (b) tan 60°

(c) cos 60° (d) sin 60°

[Delhi Gov. SQP 2022]

Ans. (d) sin 60°

Explanation: We know that,

$$\tan 30° = \frac{1}{\sqrt{3}}$$

So, $$\frac{2\tan 30°}{1+\tan^2 30°} = \frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$= \sin 60°$$

36. If $a\cot\theta + b\,\text{cosec}\theta = p$ and $b\cot + a\,\text{cosec}\theta = q$, then $p^2 - q^2$ equal to:

(a) $a^2 - b^2$ (b) $b^2 - a^2$

(c) $a^2 + b^2$ (d) $b - a$

[CBSE Term-1 Std. 2021]

Ans. (b) $b^2 - a^2$

Explanation: We have,

$$a\cot\theta + b\,\text{cosec}\theta = p$$

and $$b\cot\theta + a\,\text{cosec}\theta = q$$

So, $p^2 - q^2 = (a\cot\theta + b\,\text{cosec}\theta)^2 - (b\cot\theta + a\,\text{cosec}\theta)^2$

$= a^2\cot^2\theta + b^2\text{cosec}^2\theta + 2ab\cot\theta\,\text{cosec}\theta - (b^2\cot^2\theta + a^2\text{cosec}^2\theta + 2ab\cot\theta\,\text{cosec}\theta)$

$= a^2(\cot^2\theta - \text{cosec}^2\theta) + b^2(\text{cosec}^2\theta - \cot^2\theta)$

$= a^2(-1) + b^2(1)$ $[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

$= b^2 - a^2$

37. If $\sec\theta + \tan\theta = p$, then $\tan\theta$ is :

(a) $\frac{p^2+1}{2p}$ (b) $\frac{p^2-1}{2p}$

(c) $\frac{p^2-1}{p^2+1}$ (d) $\frac{p^2+1}{p^2-1}$

[CBSE Term-1 Std. 2021]

Ans. (b) $\frac{p^2-1}{2p}$

Explanation: We have,

$$\sec\theta + \tan\theta = p \quad ...(i)$$

We know that,

$$\sec^2\theta - \tan^2\theta = 1$$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow (\sec\theta - \tan\theta)\,p = 1$ [From (i)]

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ ...(ii)

Subtracting eq. (ii) from eq. (i), we get

$$2\tan\theta = p - \frac{1}{p}$$

$$\Rightarrow 2\tan\theta = \frac{p^2-1}{p}$$

$$\Rightarrow \tan\theta = \frac{p^2-1}{2p}$$

38. The value of if $\tan(3x - 15°) = \sqrt{3}$ is :

(a) 20° (b) 15°

(c) 25° (d) 30°

[Delhi Gov. SQP 2022]

Ans. (c) 25°

Explanation: Given, $\tan(3x - 15°) = \sqrt{3}$

$$\tan(3x - 15°) = \tan 60°$$

$$3x - 15 = 60$$

$$3x = 75$$

$$x = 25°$$

39. While eating sandwich, Chetna jokingly remarked that she can find out the value of any trigonometric ratio if just one ratio is known to her, as the sandwich is a right-angled triangle.

If 3 tan A = 4, then the value of $\frac{3 \sin A + 2 \cos A}{3 \sin A - 2 \cos A}$ is:

(a) 4 (b) $\frac{11}{15}$

(c) $\frac{7}{15}$ (d) 3

Ans. *(d) 3*

Explanation: Given, 3 tan A = 4

$$\Rightarrow \tan A = \frac{4}{3}$$

Now, $\frac{3 \sin A + 2 \cos A}{3 \sin A - 2 \cos A}$

Divide numerator and denominator by cos A

$$= \frac{\frac{3 \sin A}{\cos A} + \frac{2 \cos A}{\cos A}}{\frac{3 \sin A}{\cos A} - \frac{2 \cos A}{\cos A}} = \frac{3 \tan A + 2}{3 \tan A - 2}$$

$$= \frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2}$$

$$= \frac{4+2}{4-2} = \frac{6}{2} = 3$$

Fill in the Blanks

40. Simplest form of $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ is .... [CBSE 2020]

41. If tan A = 1, then 2 sin A cos A is .... [CBSE 2020]

Ans. 1

Explanation: Since,

$\tan A = 1,$

$\therefore A = 45°$ [$\because \tan 45° = 1$]

$\therefore 2 \sin A \cos A = 2 \times \sin 45° \times \cos 45°$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$$

42. The value of $\left(\sin^2\theta + \frac{1}{1 + \tan^2\theta}\right)$ is .... [CBSE 2020]

Ans. 1

Explanation:

$$\sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$$

$$= \sin^2\theta + \cos^2\theta = 1$$

43. Simplest form of $(1 - \cos^2 A)(1 + \cot^2 A)$ is .... [CBSE 2020]

Ans. 1

Explanation: $(1 - \cos^2 A)(1 + \cot^2 A)$

$= \sin^2 A \cdot \text{cosec}^2 A$

$= \sin^2 A \cdot \frac{1}{\sin^2 A}$

$= 1$ $\left[\because \text{cosec}\,\theta = \frac{1}{\sin\theta}\right]$

44. If $3 \sec\theta - 5 = 0$, then $\cot\theta$ is ....

45. If $\sin\theta - \cos\theta = 0$, $0 \le \theta \le 90°$ then the value of θ is ....

Ans. 45°

Explanation: $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45°$

[$\because \tan 45° = 1$]

So, $\theta = 45°$

46. cos 1° cos 2° cos 3° ...... cos 180° equals to ....

Ans. 0

Explanation:

cos 1° cos 2° cos 3°... cos 90° ... cos 180°

= cos 1° cos 2° cos 3° ....0.... cos 180°

[$\because \cos 90° = 0$]

= 0

47. If $\tan\theta = \sqrt{3}$, then sec θ is ....

Ans. 2

Explanation:

$\tan\theta = \sqrt{3}$

gives $\theta = 60°$ [$\because \tan 60° = \sqrt{3}$]

So, $\sec\theta = \sec 60° = 2$

48. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^2\theta + \cot^2\theta$ is ....

Ans. 2

Explanation:

$$\tan^2\theta + \cot^2\theta = (\tan\theta + \cot\theta)^2 - 2\tan\theta\cot\theta$$
$$= (2)^2 - 2 \times 1$$
[Given, $\tan\theta + \cot\theta = 2$]
$$= 2$$

49. Value of $\frac{2\tan^2 60°}{1+\tan^2 30°}$ is .... [CBSE 2020]

Ans. $\frac{9}{2}$

Explanation:

$$\frac{2\tan^2 60°}{1+\tan^2 30°} = \frac{2(\sqrt{3})^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{6}{\frac{4}{3}} = \frac{18}{4} = \frac{9}{2}$$

50. If $\cot\theta = \frac{7}{8}$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is .... [CBSE 2020]

Ans. $\frac{49}{64}$

Explanation: Given,

$$\cot\theta = \frac{7}{8}$$

Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

51. The value of $\frac{\cot 45°}{\sin 30° + \cos 60°}$ is .... [Delhi Gov. 2022]

Ans. 1

Explanation: we know that,

$$\tan 45° = 1$$
$$\sin 30° = \frac{1}{2}$$
and, $$\cos 60° = \frac{1}{2}$$

$$\therefore \frac{\tan 45°}{\sin 30° + \cos 60°} = \frac{1}{\frac{1}{2}+\frac{1}{2}} = 1$$

52. Given that, $\tan\theta = \frac{1}{\sqrt{3}}$, then the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is .... [Delhi Gov. 2022]

Ans. $\frac{1}{2}$

Explanation: Given, $\tan\theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \tan\theta = \tan 30°$$
$$\Rightarrow \theta = 30°$$

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{\text{cosec}^2 30° - \sec^2 30°}{\text{cosec}^2 30° + \sec^2 30°}$$

$$= \frac{2^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{2^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}}$$

$$= \frac{12-4}{12+4} = \frac{8}{16} = \frac{1}{2}$$

Assertion Reason

Direction for questions 53 to 57: In question number 53 to 57, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

53. Assertion (A): If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then the value of $x + y = 3$.

Reason (R): For any value of θ, $\sin^2\theta + \cos^2\theta = 1$

54. Assertion (A): $\sin(A + B) = \sin A + \sin B$

Reason (R): For any value of θ, $1 + \tan^2\theta = \sec^2\theta$

55. Assertion (A): $(\cos^4 A - \sin^4 A)$ is equal to $2\cos^2 A - 1$.

Reason (R): For any value of θ, $1 + \cos^2\theta = \sin^2\theta$.

Ans. (c) *Assertion (A) is true but reason (R) is false.*

Explanation: The given expression is $\cos^4 A - \sin^4 A$.

Factorising the given expression, we have

$$\cos^4 A - \sin^4 A = (\cos^2 A + \sin^2 A) \times (\cos^2 A - \sin^2 A)$$
$$= 1 \times (\cos^2 A - \sin^2 A) \quad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \cos^2 A - \sin^2 A$$
$$= \cos^2 A - (1 - \cos^2 A)$$
$$= \cos^2 A - 1 + \cos^2 A$$
$$= 2\cos^2 A - 1$$

Hence, assertion is true but reason is false.

56. Assertion (A): The value of $\sin 60° \cos 30° + \sin 30° \cos 60°$ is 1

Reason (R): $\sin 90° = 1$ and $\cos 90° = 0$

Ans. (b) *Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).*

Explanation: $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{3}{4} + \frac{1}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

We know that, $\sin 90° = 1$ and $\cos 90° = 0$

Hence, both assertion and reason are true but reason is not the correct explanation of assertion.

57. Assertion (A): In a right ΔABC, right angled at B, if $\tan A = 1$, then $2 \sin A . \cos A = 1$.

Reason (R): $\tan 45° = \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

Ans. (c) *Assertion (A) is true but reason (R) is false.*

Explanation: In ΔABC

$$\tan A = 1$$
$$\Rightarrow \tan A = \tan 45°$$
$$\Rightarrow A = 45°$$
$$\sin 45° = \frac{1}{\sqrt{2}}$$
$$\cos 45° = \frac{1}{\sqrt{2}}$$
$$2 \sin A \cos A = 2 \sin 45° \cos 45°$$
$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$
$$= \frac{2}{2}$$
$$= 1$$

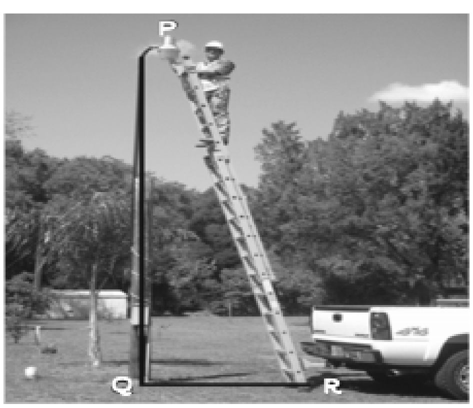
Hence, assertion is true but reason is false.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

58. *An electrician wanted to repair a street lamp at a height of 15 feet. He places his ladder such that its foot is 8 feet from the foot of the lamp post as shown in the figure below:*



(A) Find the value of cos R.

(B) Find the value of cosec P.

(C) Find the value of cot P – cosec R.

Ans. (A) We will first calculate PR by using Pythagoras theorem in ΔPQR.

So, $PR^2 = PQ^2 + QR^2$
$$= (15)^2 + 8^2$$
$$= 225 + 64$$
$$= 289 = 17^2$$
$$\Rightarrow PR = 17 \text{ feet}$$

Now, $\cos R = \frac{\text{Side adjacent to angle R}}{\text{Hypotenuse}}$

Therefore, $\cos R = \frac{QR}{PR} = \frac{8}{17}$

(C) $\cot P = \frac{\text{Base}}{\text{Perpendicular}}$

$= \frac{PQ}{QR}$

$= \frac{15}{8}$

$\text{cosec } R = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

$= \frac{PR}{PQ}$

$= \frac{17}{15}$

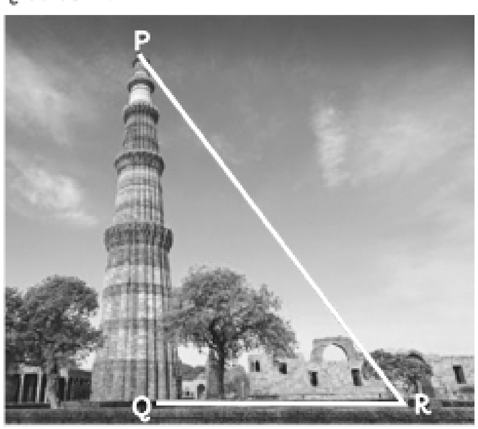
Now, $\cot P - \text{cosec } R = \frac{15}{8} - \frac{17}{15}$

$= \frac{225-136}{120}$

$= \frac{89}{120}$

59. *Built in the 13th century, the magnificent Qutub-Minar in Delhi, in red and buff sandstone is the highest tower in India. It is an architectural marvel of Ancient India. Qutub-ud-Din Aibak of Slave Dynasty laid the foundation of Minar in A.D. 1199 for the use of mu'azzin (crier) to give calls for prayer and raised the first storey, to which were added three more storeys by his successor and son-in-law, Shams-ud-Din Iltutmish (A.D. 1211-36).*

Let us take the height PQ of Qutub Minar as 72 m for ease of calculations (though actual height is 72.5 m) and distance of point R from Q as 65 m.



(A) The value of cos R is:

(a) $\frac{65}{72}$ (b) $\frac{72}{65}$

(c) $\frac{97}{65}$ (d) $\frac{65}{97}$

(B) The value of cot P is:

(a) $\frac{97}{65}$ (b) $\frac{65}{97}$

(c) $\frac{72}{65}$ (d) $\frac{65}{72}$

(C) The value of cosec P − cot P is:

(a) $\frac{12}{13}$ (b) $\frac{5}{13}$

(c) $\frac{13}{25}$ (d) $\frac{7}{12}$

(D) The value of $\sin^2 P + \cos^2 P$ is:

(a) 1 (b) − 1

(c) 0 (d) 2

(E) The value of $\frac{6}{\sin R} + \frac{13}{\sin P} - 12 \tan P$ is:

(a) 0 (b) $\frac{20}{333}$

(c) $\frac{333}{20}$ (d) 666

Ans. (A) *(d)* $\frac{65}{97}$

Explanation: We will apply Pythagoras theorem in right triangle PQR to find PR.

$PR^2 = PQ^2 + QR^2$

$= 72^2 + 65^2 = 9409 = 97^2$

$\Rightarrow PR = 97 \text{ m}$

$\cos R = \frac{\text{Side adjacent to angle R}}{\text{Hypotenuse}}$

Therefore, $\cos R = \frac{QR}{PR} = \frac{65}{97}$

(C) *(b)* $\frac{5}{13}$

Explanation: We have, ΔPQR

$\text{cosec } P = \frac{\text{Hypotenuse}}{\text{Side opposite to angle P}}$

$= \frac{PR}{QR} = \frac{97}{65}$

Also, $\cot P = \frac{\text{Side adjacent to angle P}}{\text{Side adjacent to angle P}}$

$= \frac{PQ}{QR} = \frac{72}{65}$

Now, $\text{cosec } P - \cot P = \frac{97}{65} - \frac{72}{65} = \frac{25}{65}$

$= \frac{5}{13}$

(D) *(a)* 1

Explanation: $\sin P = \frac{QR}{PR} = \frac{65}{97}$;

$$\cos P = \frac{PQ}{PR} = \frac{72}{97}$$

$$\therefore \quad \sin^2 P + \cos^2 P = \left(\frac{65}{97}\right)^2 + \left(\frac{72}{97}\right)^2$$

$$= \frac{4225+5184}{9409}$$

$$= \frac{9409}{9409} = 1$$

Alternatively,

$$\sin^2 P + \cos^2 P = 1$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

60. *Lakshaman Jhula is located 5 kilometers north-east of the city of Rishikesh in the Indian state of Uttarakhand. The bridge connects the villages of Tapovan to Jonk. Tapovan is in Tehri Garhwal district, on the west bank of the river, while Jonk is in Pauri Garhwal district, on the east bank. Lakshman Jhula is a pedestrian bridge also used by motorbikes. It is a landmark of Rishikesh. A group of Class X students visited Rishikesh in Uttarakhand on a trip. They observed from a point (P) on a river bridge that the angles of depression of opposite banks of the river are 60° and 30° respectively. The height of the bridge is about 18 meters from the river.*

Based on the above information answer the following questions.

(A) Find the distance PA.

(B) Find the distance PB.

(C) Find the width AB of the river.

OR

Find the height BQ if the angle of the elevation from P to Q be 30°.

[CBSE SQP Basic 2022]

Ans. (A) $\sin 60° = \frac{PC}{PA}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{18}{PA}$$

$$\Rightarrow \quad PA = 12\sqrt{3} \text{ m}$$

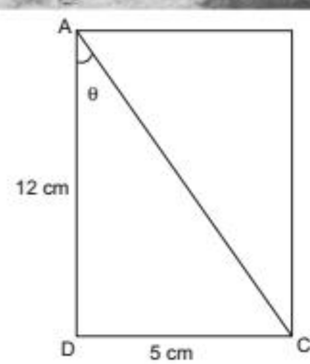
(B) $\sin 30° = \frac{PC}{PB}$

$$\Rightarrow \quad \frac{1}{2} = \frac{18}{PB}$$

$$\Rightarrow \quad PB = 36 \text{ m}$$

[CBSE Marking Scheme SQP Basic 2022]

61. *Rekha owns a rectangular-shaped gardening block, and she is planning to grow lots of fruits and vegetables in that. The block measures 12m by 5 m and the ∠CAD = θ.*



(A) Determine the value of 12 tan θ.

(B) Determine the value of $\tan^2 \theta$.

(C) Determine the value of $\frac{1-\tan^2\theta}{1+\tan^2\theta}$.

Ans. (B) $\tan^2\theta = \left(\frac{5}{12}\right)^2 = \frac{25}{144}$

(C) $\tan^2\theta = \frac{25}{144}$ [From (B)]

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{\left[1-\left(\frac{5}{12}\right)^2\right]}{\left[1+\left(\frac{5}{12}\right)^2\right]}$$

$$= \frac{1-\frac{25}{144}}{1+\frac{25}{144}}$$

$$= \frac{144-25}{144+25}$$

$$= \frac{119}{169} \text{ or } 0.704$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

62. Write the value of $\sin^2 30° + \cos^2 60°$. **[CBSE 2020]**

Ans. $\sin^2 30° + \cos^2 60° = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

63. If $\sin x + \cos y = 1$; $x = 30°$ and y is an acute angle, find the value of y. **[CBSE SQP 2020, 19]**

64. Find the value of $\sin^2 60° + 2\tan 45° - \cos^2 30°$. **[CBSE 2019]**

Ans. $\sin^2 60° + 2\tan 45° - \cos^2 30°$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2 \times 1 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{3}{4} + 2 - \frac{3}{4} = 2$$

Hence, the required value is 2.

65. If $\sin\theta = \cos\theta$, then find the value of θ. **[Delhi Gov. QB 2022]**

Ans. Given, $\sin\theta = \cos\theta$

We know that,

$$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$$

Thus, $\theta = 45°$

66. If $(1 + \cos A)(1 - \cos A) = \frac{3}{4}$, find value of sec A. **[CBSE 2019]**

67. If $\tan A = 1$ $(0° < A < 90°)$ and $\cos B = \frac{1}{\sqrt{2}}$ $(0° < B < 90°)$, then find $\cos(A + B)$.

Ans. Given, $\tan A = 1$

$\Rightarrow$ $A = 45°$

and $\cos B = \frac{1}{\sqrt{2}}$

$\Rightarrow$ $B = 45°$

So, $\cos(A + B) = \cos(45° + 45°)$

$= \cos 90° = 0$

68. Evaluate: $\frac{2\tan 45° \times \cos 60°}{\sin 30°}$. **[CBSE 2020]**

Ans. $\frac{2\tan 45° \times \cos 60°}{\sin 30°} = \frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}} = 2$

69. What is the value of $\left(\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta}\right)$? **[CBSE 2020]**

Ans. $\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta} = \frac{1}{\csc^2\theta} + \frac{1}{\sec^2\theta}$

$\left[\because \csc^2\theta = 1 + \cot^2\theta, \ \sec^2\theta = 1 + \tan^2\theta\right]$

$= \sin^2\theta + \cos^2\theta = 1$

70. Evaluate: $(\sec A + \tan A)(1 - \sin A)$, for $A = 60°$. **[CBSE 2020]**

Ans. For $A = 60°$,

$(\sec A + \tan A)(1 - \sin A)$

$= (\sec 60° + \tan 60°)(1 - \sin 60°)$

$$= \left(2+\sqrt{3}\right)\left(1-\frac{\sqrt{3}}{2}\right)$$

$$= 2+\sqrt{3}-\sqrt{3}-\frac{3}{2}$$

$$= \frac{1}{2}$$

71. If $x = a \sin\theta$ and $y = b\cos\theta$, write the value of $(b^2x^2 + a^2y^2)$. **[CBSE 2020]**

Ans. Given, $x = a\sin\theta$

and $y = b\cos\theta$

$$b^2x^2 + a^2y^2 = b^2(a^2\sin^2\theta) + a^2(b^2\cos^2\theta)$$

$$= a^2b^2[\sin^2\theta + \cos^2\theta]$$

$$= a^2b^2$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

72. If $\text{cosec}\,\theta = 2x$ and $\cot\theta = \frac{2}{x}$, find the value of $2\left[x^2 - \frac{1}{x^2}\right]$. **[CBSE 2011]**

73. If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then find $x + y$. **[CBSE SQP 2020]**

Ans.

$$x + y = 2\sin^2\theta + 2\cos^2\theta + 1$$

$$= 2(\sin^2\theta + \cos^2\theta) + 1 = 3$$

[CBSE Marking Scheme SQP 2020]

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

74. Find the value of $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ if $x = a\sin\theta$ and $y = b\cos\theta$. **[Delhi Gov. SQP 2022]**

Ans. Given, $x = a\sin\theta$ and $y = b\cos\theta$

Now, $$\frac{x^2}{a^2}+\frac{y^2}{b^2} = \frac{(a\sin\theta)^2}{a^2}+\frac{(a\cos\theta)^2}{b^2}$$

$$= \frac{a^2\sin^2\theta}{a^2}+\frac{b^2\cos^2\theta}{b^2}$$

$$= \sin^2\theta + \cos^2\theta$$

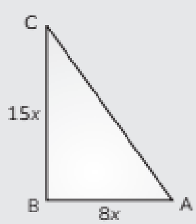
$$= 1 \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

75. Given $15\cot A = 8$, find $\sin A$ and $\sec A$. **[CBSE SQP 2020, 19]**

Ans.

$$15\cot A = 8$$

$$\cot A = \frac{8}{15}$$



$$\frac{\text{Adj}}{\text{Oppo}} = \frac{8}{15}$$

By Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$AC = \sqrt{(8x)^2+(15x)^2}$$

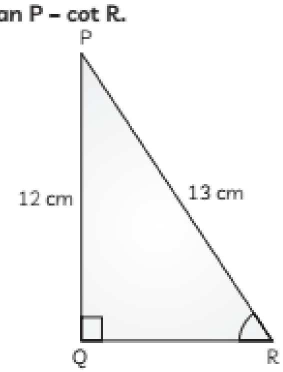
$$AC = 17x$$

$$\sin A = \frac{15}{17}$$

$$\sec A = \frac{17}{8}$$

[CBSE Marking Scheme SQP 2020]

76. Find, $\tan P - \cot R$.



[CBSE SQP 2020]

77. Prove that $1 + \frac{\cot^2\alpha}{1+\text{cosec}\,\alpha} = \text{cosec}\,\alpha$

[CBSE 2020, NCERT Exemplar]

Ans. L.H.S. $$= 1 + \frac{\cot^2\alpha}{1+\text{cosec}\,\alpha}$$

$$= 1 + \frac{\text{cosec}^2\alpha - 1}{1+\text{cosec}\,\alpha}$$

$[\because \cot^2\theta = \text{cosec}^2\theta - 1]$

$= 1 + \dfrac{(\text{cosec}\,\alpha - 1)(\text{cosec}\,\alpha + 1)}{1 + \text{cosec}\,\alpha}$

$= 1 + (\text{cosec}\,\alpha - 1)$

$= \text{cosec}\,\alpha$ = R.H.S.

Hence, proved.

78. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

[Delhi Gov. SQP 2022, CBSE 2020]

Ans. Given, $\sec^4\theta - \tan^4\theta = \tan^2\theta + \sec^2\theta$

L.H.S. $= \sec^4\theta - \tan^4\theta$

$= (\sec^2\theta)^2 - (\tan^2\theta)^2$

$= (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$

$= (\sec^2\theta + \tan^2\theta)[(1 + \tan^2\theta) - \tan^2\theta]$

$[\because \sec^2\theta = 1 + \tan^2\theta]$

$= \sec^2\theta + \tan^2\theta$

$\Rightarrow \quad \sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta$

or, $\quad \sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Hence, proved.

79. If $\cot\theta = \dfrac{7}{8}$, evaluate $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

[CBSE SQP Basic 2022]

Ans. Now,

$\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$= \dfrac{\cos^2\theta}{\sin^2\theta} = \left(\dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \cot^2\theta$

$= \left(\dfrac{7}{8}\right)^2 = \dfrac{49}{64}$

[CBSE Marking Scheme SQP Basic 2022]

80. Ⓐ **Find A and B, if $\sin(A + 2B) = \dfrac{\sqrt{3}}{2}$ and $\cos(A + B) = \dfrac{1}{2}$.** **[Diksha]**

81. If $x\cos\theta - y\sin\theta = a$, $x\sin\theta + y\cos\theta = b$, prove that $x^2 + y^2 = a^2 + b^2$.

Ans. Given, $\quad x\cos\theta - y\sin\theta = a$ ...(i)

and $\quad x\sin\theta + y\cos\theta = b$ ...(ii)

Squaring and adding equations (i) and (ii), we get

$(x\cos\theta - y\sin\theta)^2 + (x\sin\theta + y\cos\theta)^2 = a^2 + b^2$

$\Rightarrow \quad x^2\cos^2\theta + y^2\sin^2\theta - 2xy\sin\theta\cos\theta + x^2\sin^2\theta + y^2\cos^2\theta + 2xy\sin\theta\cos\theta = a^2 + b^2$

$\Rightarrow \quad x^2(\cos^2\theta + \sin^2\theta) + y^2(\sin^2\theta + \cos^2\theta) = a^2 + b^2$

$\Rightarrow x^2 + y^2 = a^2 + b^2 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

Hence, proved.

82. Ⓐ **If $x = a\cos^3\theta$, $y = b\sin^3\theta$, prove that $\left(\dfrac{x}{a}\right)^{2/3} + \left(\dfrac{y}{b}\right)^{2/3} = 1.$** **[Diksha]**

83. Evaluate:

$(\sin^4 60° + \sec^4 30°) - 2(\cos^2 45° - \sin^2 90°)$

Ans. $(\sin^4 60° + \sec^4 30°) - 2(\cos^2 45° - \sin^2 90°)$

$= \left[\left(\dfrac{\sqrt{3}}{2}\right)^4 + \left(\dfrac{2}{\sqrt{3}}\right)^4\right] - 2\left[\left(\dfrac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$

$= \left(\dfrac{9}{16} + \dfrac{16}{9}\right) - 2\left(\dfrac{1}{2} - 1\right)$

$= \left(\dfrac{81 + 256}{144}\right) - 2\left(-\dfrac{1}{2}\right)$

$= \dfrac{337}{144} + 1$

$= \dfrac{337 + 144}{144}$

$= \dfrac{481}{144}$

84. If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$. **[Diksha]**

Ans. Given, $a\cos\theta - b\sin\theta = c$

On squaring both sides, we get

$(a\cos\theta - b\sin\theta)^2 = c^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$

$\Rightarrow a^2(1 - \sin^2\theta) + b^2(1 - \cos^2\theta) - 2ab\cos\theta\sin\theta = c^2$

$\left[\begin{array}{l}\because \cos^2\theta = 1 - \sin^2\theta \\ \sin^2\theta = 1 - \cos^2\theta\end{array}\right]$

$\Rightarrow a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\cos\theta\sin\theta = c^2$

$$\Rightarrow a^2 \sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta = a^2 + b^2 - c^2$$

$$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$$

$$\Rightarrow a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}.$$

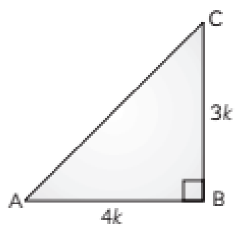
Hence, proved.

85. **Simplify $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$.** **[CBSE 2013]**

86. **If $\tan A = \frac{3}{4}$, find the value of $\frac{1}{\sin A} + \frac{1}{\cos A}$.** **[CBSE SQP 2020]**

Ans. Given, $\tan A = \frac{3}{4}$

Then, let perpendicular = $3k$ and base = $4k$



Now, by applying Pythagoras theorem in ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= (4k)^2 + (3k)^2$$
$$= 16k^2 + 9k^2$$
$$= 25k^2$$

$\therefore$ $AC = 5k$

$\therefore$ $\sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

Then, $\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{1}{\frac{3}{5}} + \frac{1}{\frac{4}{5}} = \frac{5}{3} + \frac{5}{4}$

$$= \frac{20+15}{12} = \frac{35}{12}$$

87. **If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0° < \theta < 90°$, find the value of θ.** **[CBSE SQP 2020]**

Ans. $\sqrt{3}\sin\theta = \cos\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

[CBSE Marking Scheme SQP 2020]

Explanation : Given, $\sqrt{3}\sin\theta - \cos\theta = 0$

Then, $\sqrt{3}\sin\theta = \cos\theta$

$\Rightarrow$ $\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan\theta = \frac{1}{\sqrt{3}}$

Then, $\tan\theta = \tan 30°$

$\left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$

$\therefore$ $\theta = 30°$

Hence, the value of θ is 30°.

88. **If $\tan\theta = \frac{3}{4}$, find the value of $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$.** **[CBSE 2020]**

89. **If $\tan\theta = \sqrt{3}$, find the value of $\frac{2\sec\theta}{1+\tan^2\theta}$.** **[CBSE 2020]**

90. **If $K + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$, find the value of K.** **[British Council 2022]**

Ans. $K + 1 = \sec^2\theta(1 + \sin\theta)(1 - \sin\theta)$

$= \sec^2\theta(1 - \sin^2\theta)$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$= \sec^2\theta\cos^2\theta$

$[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \frac{1}{\cos^2\theta}.\cos^2\theta$

$K + 1 = 1$

$K = 0$

91. Prove that $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = 2\tan x \sec x$.

[British Council 2022]

Ans.

$$\text{L.H.S.} = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$$

$$= \frac{1+\sin x - 1 + \sin x}{(1-\sin^2 x)}$$

$$= \frac{2\sin x}{(1-\sin^2 x)}$$

$$= \frac{2\sin x}{\cos^2 x}$$

$$= \frac{2\sin x}{\cos x} \times \frac{1}{\cos x}$$

$$= 2\tan x \sec x$$

= R.H.S.

Hence, Proved.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

92. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$. [CBSE SQP Basic 2022]

Ans. $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow \sin\theta\cos\theta = 1$

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

[CBSE Marking Scheme SQP Basic 2022]

93. Prove that: $\frac{\sin\theta - \cos\theta + 1}{\cos\theta + \sin\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$

[CBSE 2020]

Ans. $\text{L.H.S.} = \frac{\sin\theta - \cos\theta + 1}{\cos\theta + \sin\theta - 1}$

$$= \frac{\tan\theta - 1 + \sec\theta}{1 + \tan\theta - \sec\theta}$$

[Dividing numerator and denominator by $\cos\theta$]

$$= \frac{(\tan\theta + \sec\theta) - 1}{1 + \tan\theta - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{1 + \tan\theta - \sec\theta}$$

$\left[\because \sec^2\theta - \tan^2\theta = 1\right]$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{1 + \tan\theta - \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{1 + \tan\theta - \sec\theta}$$

$= \tan\theta + \sec\theta$

$$= (\tan\theta + \sec\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta - \tan\theta} = \text{R.H.S.}$$

Hence, proved.

94. Prove that :

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$$

[CBSE SQP Basic 2022]

Ans. $\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{(\tan^2\theta + \tan\theta + 1)}{\tan\theta}$$

$= \tan\theta + 1 + \cot\theta = 1 + \tan\theta + \cot\theta$

$$= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\ \cos\theta}$$

$$= 1 + \frac{1}{\sin\theta\ \cos\theta} = 1 + \sec\theta\ \mathrm{cosec}\ \theta$$

[CBSE Marking Scheme SQP Basic 2022]

95. Prove that: $\dfrac{\cot\theta + \mathrm{cosec}\ \theta - 1}{\cot\theta - \mathrm{cosec}\ \theta + 1} = \dfrac{1 + \cos\theta}{\sin\theta}$

[CBSE 2020]

96. Prove that:

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0.$

[CBSE 2020]

Ans. We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, $(\sin^2\theta + \cos^2\theta)^2 = 1^2$

$\Rightarrow \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$

i.e., $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$ ...(i)

Also, $(\sin^2\theta + \cos^2\theta)^3 = 1^3$

$\Rightarrow \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$(\sin^2\theta + \cos^2\theta) = 1$

$\Rightarrow \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta\,(1) = 1$

i.e., $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ ...(ii)

now,

LHS $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2(1 - 3\sin^2\theta\cos^2\theta) - 3(1 - 2\sin^2\theta\cos^2\theta) + 1$

$= 2 - 3 + 1$

$= 0$

Hence, proved.

97. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$. [CBSE 2020, 19]

98. Prove that:

$(\sin^4\theta - \cos^4\theta + 1)\ \mathrm{cosec}^2\theta = 2$. [CBSE 2020].

Ans. L.H.S. $= (\sin^4\theta - \cos^4\theta + 1)\ \mathrm{cosec}^2\theta$

$= [(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) + 1]\ \mathrm{cosec}^2\theta$

$= [(1)(\sin^2\theta - \cos^2\theta) + 1]\ \mathrm{cosec}^2\theta$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= [\sin^2\theta + (1 - \cos^2\theta)]\ \mathrm{cosec}^2\theta$

$= (\sin^2\theta + \sin^2\theta)\ \mathrm{cosec}^2\theta$

$= (2\sin^2\theta)\ \mathrm{cosec}^2\theta$

$= 2\sin^2\theta \times \dfrac{1}{\sin^2\theta}$ $\left[\because \mathrm{cosec}\ \theta = \dfrac{1}{\sin\theta}\right]$

$= 2 \times 1$

$= 2 =$ R.H.S.

Hence, proved.

99. **Prove the following that:**

$$\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\ \mathrm{cosec}\theta - 2\sin\theta\cos\theta$$

[CBSE SQP Std. 2022]

Ans. LHS : $\dfrac{\sin^3\theta/\cos^3\theta}{1+\sin^2\theta/\cos^2\theta} + \dfrac{\cos^3\theta/\sin^3\theta}{1+\cos^2\theta/\sin^2\theta}$

$$= \frac{\sin^3\theta/\cos^3\theta}{(\cos^2\theta+\sin^2\theta)/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{(\sin^2\theta+\cos^2\theta)/\sin^2\theta}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$

$= \sec\theta\ \mathrm{cosec}\theta - 2\sin\theta\cos\theta$

= RHS

[CBSE Marking Scheme SQP Std. 2022]

100. **Prove that:** $\dfrac{\tan\theta}{1-\tan\theta} - \dfrac{\cot\theta}{1-\cot\theta} = \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

[CBSE 2019]

Ans. L.H.S. $= \dfrac{\tan\theta}{1-\tan\theta} - \dfrac{\cot\theta}{1-\cot\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1 - \frac{\sin\theta}{\cos\theta}} - \frac{\frac{\cos\theta}{\sin\theta}}{1 - \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\sin\theta}{\cos\theta - \sin\theta} - \frac{\cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta}{\cos\theta - \sin\theta}$$

$= \frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta}$ or $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

= R.H.S.

Hence, proved.

101. **If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.** **[CBSE 2019]**

Ans. Given: $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$

Multiplying both sides by $(\sqrt{2} + 1)$, we get

$\Rightarrow (\sqrt{2} + 1)\sin\theta = (\sqrt{2}+1)(\sqrt{2}-1)\cos\theta$

$\Rightarrow (\sqrt{2} + 1)\sin\theta = (2 - 1)\cos\theta$

$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = \cos\theta$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Hence, proved.

102. **Prove that:**

$(\sin\theta + 1 + \cos\theta)(\sin\theta - 1 + \cos\theta)\sec\theta\,\mathrm{cosec}\,\theta = 2$ **[CBSE 2019]**

103. **Prove that: $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\,\mathrm{cosec}\,\theta$**

[CBSE 2019]

Ans. L.H.S. $= \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$

$= \frac{(\sqrt{\sec\theta - 1})^2 + (\sqrt{\sec\theta + 1})^2}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}}$

$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{\sec^2\theta - 1}}$

$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}}$ $[\because \tan^2\theta = \sec^2\theta - 1]$

$= \frac{2\sec\theta}{\tan\theta}$

$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$

$= \frac{2}{\sin\theta} = 2\,\mathrm{cosec}\,\theta$ = R.H.S.

Hence, proved.

104. **If $4\tan\theta = 3$, evaluate $\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}$**

[CBSE 2018]

Ans. Given, $4\tan\theta = 3$

$\Rightarrow \tan\theta = \frac{3}{4}$

Squaring both sides, we get

$\tan^2\theta = \frac{9}{16}$

$\Rightarrow \sec^2\theta - 1 = \frac{9}{16}$

$[\because \tan^2\theta = \sec^2\theta - 1]$

$\Rightarrow \sec^2\theta = \frac{25}{16}$

$\Rightarrow \sec\theta = \frac{5}{4}$

Now, $\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}$

[Dividing the numerator and denominator by $\cos\theta$.]

$= \frac{\frac{4\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{4\sin\theta + \cos\theta - 1}{\cos\theta}}$

$= \frac{4\tan\theta - 1 + \sec\theta}{4\tan\theta + 1 - \sec\theta}$

$= \frac{4 \times \frac{3}{4} - 1 + \frac{5}{4}}{4 \times \frac{3}{4} + 1 - \frac{5}{4}}$

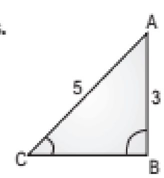
$= \frac{2 + \frac{5}{4}}{4 - \frac{5}{4}}$

$= \frac{13}{11}$

105. **If $\sin\theta = \frac{3}{5}$, then find $\tan\theta - \sec\theta$.**

[Delhi Gov. SQP 2022]

Ans.



Given, $\sin\theta = \frac{3}{5}$

In the ΔABC

$AC^2 = AB^2 + BC^2$

$(5)^2 = (3)^2 + BC^2$

$BC^2 = 25 - 9$

$= 16$

$BC = 4$

Now, $\tan\theta = \frac{AB}{BC} = \frac{3}{4}$

$$\sec\theta = \frac{AC}{BC} = \frac{5}{4}$$

So, $$\tan\theta - \sec\theta = \frac{3}{4} - \frac{5}{4}$$

$$= \frac{-2}{4} = \frac{-1}{2}$$

106. Prove the following identity:

$$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

107. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, show that $q(p^2 - 1) = 2p$. [CBSE 2012]

Ans. L.H.S. $= q(p^2 - 1)$

$= (\sec\theta + \text{cosec}\,\theta)\,[(\sin\theta + \cos\theta)^2 - 1]$

$= (\sec\theta + \text{cosec}\,\theta)\,[(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) - 1]$

$= (\sec\theta + \text{cosec}\,\theta)\,[(1 + 2\sin\theta\cos\theta) - 1]$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$

$$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right)(2\sin\theta\cos\theta)$$

$= 2(\sin\theta + \cos\theta) = 2p$

$=$ R.H.S.

Hence, proved.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

108. Prove that:

$$\frac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{(\sec^3\theta - \text{cosec}^3\theta)} = \sin^2\theta\cos^2\theta$$

[CBSE 2019]

Ans. $$\text{L.H.S.} = \frac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{(\sec^3\theta - \text{cosec}^3\theta)}$$

$$= \frac{\left(1 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)(\sin\theta - \cos\theta)}{\frac{1}{\cos^3\theta} - \frac{1}{\sin^3\theta}}$$

$$= \frac{\frac{(\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta)(\sin\theta - \cos\theta)}{\sin\theta\cos\theta}}{\frac{\sin^3\theta - \cos^3\theta}{\cos^3\theta\sin^3\theta}}$$

$$= \frac{(\sin\theta\cos\theta + 1)(\sin\theta - \cos\theta)\cos^2\theta\sin^2\theta}{(\sin^3\theta - \cos^3\theta)}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \frac{(1 + \sin\theta\cos\theta)(\sin\theta - \cos\theta)\cos^2\theta\sin^2\theta}{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}$$

$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$$= \frac{(1 + \sin\theta\cos\theta)}{(1 + \sin\theta\cos\theta)} \times \cos^2\theta\sin^2\theta$$

$= \sin^2\theta\cos^2\theta =$ R.H.S.

Hence, proved.

109. Prove that:

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$$

[CBSE 2019]

Ans. $$\text{L.H.S.} = \frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta}$$

$$= \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$$

$[\because (a^3 - b^3) = (a - b)(a^2 + ab + b^2)]$

$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

$= \tan\theta + 1 + \cot\theta$

$$= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$= 1 + \frac{1}{\sin\theta\cos\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 + \sec\theta\,\text{cosec}\,\theta$

= R.H.S.

Hence, proved.

110. Prove that: $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

[CBSE 2018]

Ans. L.H.S. $= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \frac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$

$= \frac{\sin A\,[1 - 2(1 - \cos^2 A)]}{\cos A\,(2\cos^2 A - 1)}$

$[\because \sin^2 A + \cos^2 A = 1]$

$= \frac{\tan A\,(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$

$= \tan A$ = R.H.S.

Hence, proved.